(Time: 3 Hours)                                                        (Total Marks: 80)

**N.B.**   (1) Question No. 1 is compulsory.
           (2) Answer any three questions from Q.2 to Q.6.
           (3) Use of Statistical Tables permitted.
           (4) Figures to the right indicate full marks.

**Q1.** (a)  Find the Laplace transform of $t\, e^{-t} \cosh 2t$                                    [05]

(b) If $u = -r^3 \sin 3\theta$ find the analytic function $f(z)$ whose real part is u.       [05]

(c) Calculate the Spearman's rank correlation coefficient R

| x | 85 | 74 | 85 | 50 | 65 | 78 | 74 | 60 | 74 | 90 |
|---|----|----|----|----|----|----|----|----|----|----|
| y | 78 | 91 | 78 | 58 | 60 | 72 | 80 | 55 | 68 | 70 |

[05]

(d) Find inverse Laplace transform of $\frac{1}{s} \log\left(1 + \frac{1}{s^2}\right)$.                [05]

**Q2.** (a)  Evaluate by using Laplace transform of $\int_0^\infty e^{-2t}\, \frac{\cos 2t \sin 3t}{t}\, dt$.       [06]

(b) Find the value of k if the function $f(x) = k\, x\, e^{\frac{-x}{3}}$, $x > 0$
$$f(x) = 0 \qquad 0 \le x.$$
Is a probability density function. find mean and variance.          [06]

(c) Obtain the Fourier series to represent $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$

Hence show that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \ldots$          [08]

**Q3.** (a) Find the analytic function whose real part is $u = e^{2x}(x \cos 2y - y \sin 2y)$.    [06]

(b) Obtain the Fourier series to represent $f(x) = x - x^2$, $-1 \le x \le 1$.       [06]

(c) Using convolution theorem Find inverse Laplace transform of $\frac{(s+3)^2}{(s^2 + 6s + 18)^2}$.   [08]

**Q4.** (a) Obtain the half range cosine series of $f(x) = x(\pi - x)$ in $(0, \pi)$

Hence show that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \ldots$       [06]

(b) Find the lines of regression and coefficient of correlation for the data       [06]

| x | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
|---|----|----|----|----|----|----|----|----|
| y | 67 | 68 | 65 | 66 | 72 | 72 | 69 | 71 |

(c) Evaluate by using Laplace transform of $\int_0^\infty e^{-t} \left(\int_0^t u^2 \sin hu \cos hu\, du\right) dt$   [08]

**Q5.** (a) Find the orthogonal trajectories of family of curves $e^{-x} \cos y + x\, y = \alpha$
where $\alpha$ is the real constant in the x y - plane.                      [06]

(b)  A random variable x has the probability distribution    [06]

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P ( x =x ) | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{6}$ |

Find the moment generating function about origin. also find mean and variance.

(c) Fit a second degree parabolic curve to the following:    [08]

| X year | 1965 | 66 | 67 | 68 | 69 | 70 | 71 | 72 |
|--------|------|-----|-----|-----|-----|-----|-----|-----|
| Y profit | 125 | 140 | 165 | 195 | 200 | 215 | 220 | 230 |

Also estimate the profit in 1973

Q6. (a)  Find inverse Laplace transform of $\frac{(2s^2 -6s +5)}{(s^3 -6 s^2 +11 s-6)}$    [06]

(b) Show that the function $v = e^x$ (x sin y + y cos y) satisfies Laplace equation
And find its corresponding analytic function and its harmonic conjugate.    [06]

(c) A random variable X has the probability function    [08]

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| P( X = x) | K | 2K | 3K | $K^2$ | $K^2 + K$ | $2K^2$ | $4K^2$ |

Find  k , p ( X < 5 ) , p ( x > 3), P ( 0 ≤ X ≤ 5 ).

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**(3 hours)**                                                    **Total Marks: 80**

N.B.    1. Question **No. 1** is compulsory

2. Attempt any **three** questions from remaining five questions

3. Assume suitable data if **necessary** and justify the assumptions

4. Figures to the **right** indicate full marks

| | | |
|---|---|---|
| Q1 | A | What are universal logic gates? Why are they called so? Explain with a suitable example | 05 |
| | B | Explain the functioning of D and T flip-flops along with their Truth table | 05 |
| | C | Differentiate between Hardwired control unit and Micro programmed control unit | 05 |
| | D | List and describe the key characteristics of memory? | 05 |
| Q2 | A | Using booths algorithm multiply 3 x -2 along with its flow chart do write appropriate comments for each operation. | 10 |
| | B | Draw the flow chart for Restoring division algorithm and Perform $6 \div 3$ | 10 |
| Q3 | A | Explain Multiplexer & Demultiplexerx (IC level description only) | 10 |
| | B | Discuss the different ways in which data can be accessed in memory using addressing modes. | 10 |
| Q4 | A | Explain Micro instruction format and write a micro program for the instruction ADD $R_1$ , $R_2$ | 10 |
| | B | Explain Hardwired Control Unit and the various design methods associated with it. | 10 |
| Q5 | A | Explain different memory Mapping Techniques | 10 |
| | B | Describe Interleaved memory | 05 |
| | C | What do you mean by cache coherence | 05 |
| Q6 | A | Explain Instruction pipelining and describe the hazards associated with it | 10 |
| | B | Explain Flynn's Classification. | 10 |

**70846**

Time:3 Hrs                                                                           Marks:100

N.B. : (1) Question Number 1 is compulsory
2)Solve any three questions from the remaining questions
3)Make suitable assumptions if needed
4)Assume appropriate data whenever required. State all assumptions
clearly.

1. a.   Define the following with suitable example                                   5
a)  Power Set  b) Group c) Euler Graph d) Existential Quantifier

b.   Construct the Truth Table and check if the following statement is tautology.    5
$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

c.   For all sets A, B and C show that                                               5
$A \times (B \cap C) = (A \times B) \cap (A \times C)$

d.   Prove by mathematical induction that                                           5
$1.1! + 2.2! + 3.3! + -- + n.n! = (n+1)! -1$

2   a   Define Equivalence Relation. Let A be a set of integers, Let R be a Relation on    8
AXA defined by (a,b) R (c,d) if and only if a+d = b+c. Prove that R is an
Equivalence Relation

b.   Let A={a, b, c, d}   Find Transitive Closure of R represented by $M_R$          8
using Warshall's algorithm.

$$M_R = \begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

c   Prove that the set A=(0,1,2,3,4,5) is a finite Abelian group under Addition modulo 6   4

3   a   Let f,g,h be functions on real numbers R defined as follows:                 8
f(x) = 2x+5,    g(x) = 5x + 3,    h(x) = 3x
Find:  1) g o f   2) g o h   3) f o g o h   4) g o f o h

b   Give the exponential generating function for the sequences                8
    1) $\{1,1,1\ldots\}$
    2) $\{1,2,3,4,\ldots\}$
    3) $\{1, a, a^2, a^3, \ldots\}$

c   Determine whether the following graphs are isomorphic. Justify your answer.   4
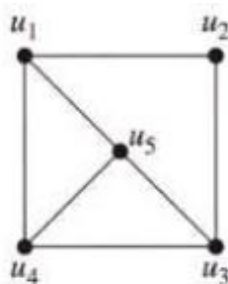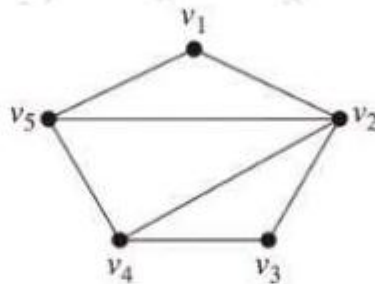


        G1           G2

4   a   A Function                                                               8
    $f: R - \{(\tfrac{2}{5})\} \to R - \{4/5\}$ is defined as  $f(x) = (4x+3)/(5x-2)$
    Prove that f is Bijective and find the rule for $f^{-1}$

   b   Show that (2,5) encoding function $e:B^2 \to B^5$ defined by              8
    e (00) = 00000
    e (01) = 01110
    e (10) = 10101
    e (11) = 11011
    is a group code.

   c   Find the number of positive integers n where $1 \le n \le 100$ and n is not divisible  by 2,3 4
    or 5.

5    a    Define Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit.        8
Determine if    the following diagram has Euler Path. Euler Circuit, Hamiltonian Path
and Hamiltonian Circuit and state the path /circuit.
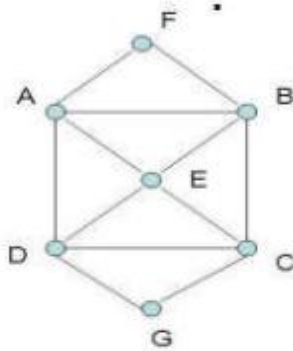


    b    State and explain the extended Pigeonhole principle. How many friends must you    8
have to guarantee that at least five of them will have birthdays in the same month.

    c    Find the complement of each element in $D_{42}$                                      4

6.    a    Draw the Hasse Diagram of $D_{72}$ and check whether it is a Lattice.            8

    b    Find the complete solution of $a_n + 2a_{n-1} = n+3$ for $n \geq 1$ with $a_0=3$      8

    c    Define the following with suitable examples.                            4
a)Maximal and Minimal Element    b) Partition of a set    c) Sub Lattice d) Injective
Function

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

**Time : 3 Hours**                                                              **Marks: 80**

**N.B:** 1) Question **number 1** is compulsory.

2) Attempt **any three** out of the remaining.

3) Assume suitable data if **necessary** and justify the assumptions.

4) Figures to the **right** indicate full marks.

Q.1 Attempt **any four**                                                        20

   a) Give difference between random scan display and raster scan display.

   b) Define Aliasing, Describe different antialiasing techniques.

   c) Compare DDA and BRESENHAM line drawing algorithm.

   d) Explain point clipping algorithm.

   e) Give fractal dimension for KOCH curve.

Q.2 a) Derive formula for mid-point circle algorithm.                            10

   b) Given a line AB where A(0,0) and B(1,5) calculate all the points of line AB using
      DDA algorithm.                                                    10

Q.3 a) With neat diagram explain Composite transformation.                       10

   b) Given a triangle ABC where A(0,0), B(-10,-10) and C(10,-10) rotate the given triangle
      ABC 180 degree in anti-clockwise direction. Find out the new co-ordinate of triangle
      ABC after rotation.                                               10

Q.4 a) With neat diagram explain window to viewport coordinate transformation.   10

   b) With neat diagram explain Sutherland Hodgman polygon clipping algorithm.   10

Q.5 a) Define projection, with neat diagram describe planar geometric projection. 10

   b) Describe properties of BEZIER curve.                                   10

Q.6 a) Describe various principles of traditional animation.                     10

   b) Write short note on Depth buffer algorithm.                            10

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**(3 Hours)**        **Total Marks: 80**

**N.B: (1) Question No. 1 is compulsory**
     **(2) Attempt any three questions out of the remaining five questions**
     **(3) Figures to the right indicate full marks**
     **(4) Make suitable assumptions wherever necessary with proper justifications**

| | | | |
|---|---|---|---|
| Q1 | A | Explain the concept of Abstract Data Type with an example. | [05] |
| | B | What are the disadvantages of representing a linear queue using an array? How are they overcome? | [05] |
| | C | Given an array based stack implemented with a maximum size of 4, perform the following operations in sequence: Push(12), Push(25), Push(33), Pop(), Push(47), Push(51), Push(66). Now, based on the sequence of operations, apply your knowledge to: | [05] |

       i.    Determine the state of the stack after each operation.
       ii.   Identify if any overflow or underflow conditions occur during these operations. If so, at which step do they occur?
       iii.  Write conditions to check for stack overflow and underflow.

| | | | |
|---|---|---|---|
| | D | Write an algorithm to perform binary search on a given set of 'n' numbers. | [05] |
| Q2 | A | Consider two different orders of inserting the elements 40, 20, 60, 10, 30, 50, 70 into an empty Binary Search Tree (BST): | [10] |

       i.    Insert the elements in the given order.
       ii.   Insert the elements in reverse order.

Construct both BSTs and compare their heights.

| | | | |
|---|---|---|---|
| | B | Write a program in C to create a Singly linked list. Include functions to insert element at the second last position and display every alternate element of the list. | [10] |
| Q3 | A | Explain Depth First search and Breadth First search graph traversal techniques with example. | [10] |
| | B | Given the values {11, 9, 62, 51, 6, 99, 16, 9, 58, 47}, a hash table of size 10 and a hash function h(k) = k mod 10, show the resulting table after inserting the values in the given order using Linear probing technique. | [10] |

Q4 A Given the set of characters and frequencies:       **[10]**
    M: 4, N: 8, O: 16, T: 32, E: 64.
    Construct the Huffman tree and write the binary code for each symbol and
    encode the string "MOMENT".

  B Write a program in C to implement Circular queue using an array.   **[10]**

Q5 A Explain the key differences between a singly linked list, a doubly linked **[10]**
    list, and a circular linked list. Use diagrams to show the structure of each
    type and discuss the advantages and disadvantages of each.

  B Write a program in C to evaluate a postfix expression.     **[10]**

Q6 A Write a program in C to remove all occurrences of a specific value from a **[10]**
    given doubly linked list.

  B Perform a series of insertions with the elements 9, 15, 19, 8, 7, 13, 10, 25, **[10]**
    30, 14. Show all the rebalancing steps required to keep the AVL tree
    balanced.

———————————————