Paper / Subject Code: N10411 / Applied Mathematics-I

Applied Mathematics-I
Revised Course (NEP-2020)
(Dec 2024)

Marks-60
Durations-02 Hours

Question 1 is compulsory.
Attempt any 3 from questions 2 to 6.
Scientific Calculator is allowed to use.

1. Attempt any **Five** questions. **(Compulsory Problem)**

(a) Find all values of $(1+i)^{1/3}$. **3**

(b) Prove that: $tanh(log\sqrt{3}) = 0.5$ **3**

(c) If $u = (1-2xy+y^2)^{-1/2}$, prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$. **3**

(d) Find n^{th} derivative of $y = e^{ax}\sin^2 x$. **3**

(e) Show that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary and hence find A^{-1}. **3**

(f) Using Newton Raphson's Method, find an iterative formula for $\sqrt[3]{150}$. **3**

2. (a) If $H = f(y-z, z-x, x-y)$, then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$. **4**

(b) Prove that $\log(e^{i\alpha} + e^{i\beta}) = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$. **5**

(c) Apply the Gauss Seidel method, to solve the following system of linear equations up to the two iterations. **6**
$15x + 3y - 2z = 85,\ x - 2y + 8z = 5, 2x + 10y + z = 51$.

3. (a) Discuss the maxima and minima of $(x^2 + y^2 + 8x + 6y + 6)$. **4**

(b) Solve the equation $x^4 + x^3 + x^2 + x + 1 = 0$. **5**

(c) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial x^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$. **6**

4. (a) Prove that $sech^{-1}(\sin\theta) = \log(\cot\frac{\theta}{2})$. **4**

(b) Prove that $cos^6\theta = \frac{1}{32}(cos6\theta + 6cos4\theta + 15cos2\theta + 10)$. **5**

(c) Find nonsingular matrices P & Q such that PAQ is in normal form and hence find rank of the matrix A for the following matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. **6**

5. (a) Find a, b, c & A^{-1} if $A = \frac{1}{9}\begin{bmatrix} 8 & -4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$ is orthogonal. **4**

(b) State & prove Euler's theorem on homogeneous functions with two independent variables. **5**

(c) If $y = e^{tan^{-1}x}$ then prove that $(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$. **6**

6. (a) Find n^{th} derivative of $y = \frac{x}{1+3x+2x^2}$. **4**

(b) Investigate for what values of λ and μ the system of linear equations $x + y + z = 6;\ x + 2y + 3z = 10;$ $x + 2y + \lambda z = \mu$ will have (i) No solution, (ii) Unique solution, (iii) Infinite number of solutions. **5**

(c) Find the three sets of iterative solutions of the following equations by Gauss Jacobi method: **6**
$5x - y + z = 10\ ;\ 2x + 4y = 12\ ;\ x + y + 5z = -1$.

.....

Duration: 1 ½ hours **[Max Marks:45]**

N.B. : (1) Question No 1 is Compulsory.
(2) Attempt any two questions out of the remaining four.
(3) Assume suitable data, if required and state it clearly.

1 Attempt any FIVE **[15]**

(a) What is metastable state? What is the role of metastable state in LASER?
(b) An optical fiber has an acceptance angle of 30° and core refractive index of calculate refractive index of cladding?
(c) Explain why an excessively thin film appears black in reflected light?
(d) Show that the divergence of a curl of a vector is zero.
(e) State the properties of matter waves.
(f) An electron is bound in an 1D potential well of width 2 A° but of infinite height. Find its energy values in the ground state and in first excited state. (Given: Planck constant is 6.634×10^{-34} J-Sec)
(g) Write Fermi Dirac distribution function and explain the terms involved in it

2 (a) With the neat energy level diagram, explain the construction and working of He Ne Gas laser. **[05]**
(b) Derive an expression for numerical aperture of a step index optical fiber. **[05]**
(c) In a Newton's ring arrangement, a drop of liquid of refractive index 1.33 is placed in between the lens and plate. The diameter of 9^{th} dark ring is found to be 0.58 cm. Obtain the radius of curvature, if the wavelength of light is 6000 A°. **[05]**

3 (a) What is gradient? If $\emptyset = 3(x^2y - y^2x)$, find $\nabla\emptyset$ at the point (1, -2,-1). **[05]**
(b) Prove that electron cannot reside inside the nucleus of an atom. **[05]**
(c) Predict the effect on the electrical properties of a silicon at room temperature if every millionth silicon atom is replaced by an atom of indium. **[05]**
Given: Concentration of silicon atoms are 5×10^{28} /m^3
intrinsic conductivity of silicon is 4.4×10^{-4} /Ω-m
mobility of holes is 0.048 m^2 / volt-sec.

4	(a)	Show that for an intrinsic semiconductor, fermi level lies midway of conduction band and valence band.	[05]
	(b)	With the experimental set up of Newton's ring, explain the determination of refractive index of a liquid.	[05]
	(c)	An electron and a photon each have a wavelength of 2A°.What are their momentum and energies?	[05]
5	(a)	Compare Step Index fiber and Graded Index Fiber.	[05]
	(b)	State and explain Maxwell's fourth equation.	[05]
	(c)	What is Lidar technology and how does it work?	[05]

(3 Hours) **Max. Marks:- 80**

Note: - 1) Question No1. is compulsory.
2) Attempt any Three out of remaining.

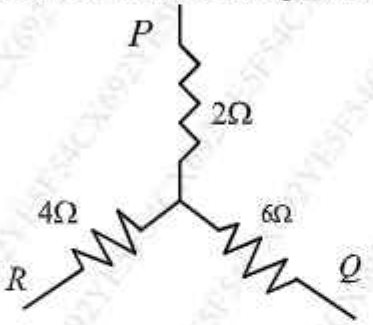
Q1 Attempt **any FOUR** **(20)**

a) Derive an emf equation of a transformer **(5)**

b) Two wattmeters are used to measure power in a three-phase balanced load. The wattmeter readings are 8.2 kW and 7.5 kW. Calculate total power, power factor and reactive power. **(5)**

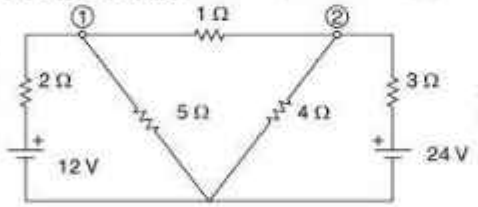
c) In a circuit a voltage of 200< 40°V is applied to a two-element circuit. A current of 20<-20°A is flowing through it. Find elements of impedance. **(5)**

d) Convert star into its equivalent delta. **(5)**

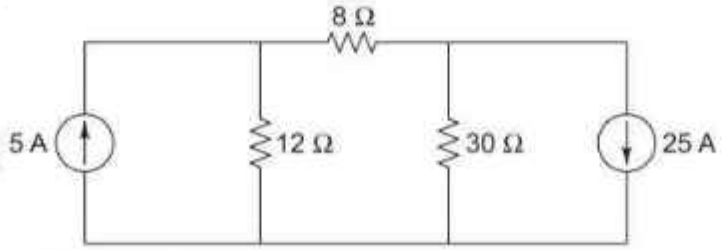


e) Two currents are meeting at a point. Find the resultant current. **(5)**
i1= 100 Sin(ωt)A, i2= 25 Cos(ωt - $\frac{\pi}{3}$)A.

Q2 a) Obtain the current through the 1Ω resistor using nodal analysis method for the circuit. **(10)**

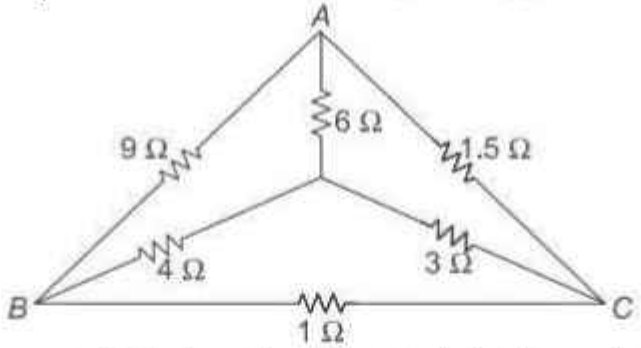


b) **(10)**



Find current through 8 ohm by mesh analysis.

Q3 a) Find equivalent resistance between A & B in the network shown. **(10)**



b) A current of 5 A flows through a non-inductive resistor in series with a choke coil when supplied at 250 V, 50 Hz. If the voltage drops across the coil and noninductive resistor are 200 V and 125 V respectively, calculate the resistance and inductance of the impedance coil, value of non-inductive resistor and power drawn by the coil. **(10)**

Q4 a) Voltage and current in an ac circuit are given by **(10)**

$v = 200 \sin 377\ t$ $\quad i = 8 \sin (377\ t - \pi/6)$

Determine true power, reactive power and apparent power drawn by the circuit

b) Explain working principle of three phase induction motor and mention its types. **(5)**

c) Explain working of variable reluctance stepper motor. **(5)**

Q5 a) A symmetrical three-phase 400 V system supplies a basic load of 0.8 lagging power factor and is connected in star. If the line current is 34.64 A, find the (i) impedance, (ii) resistance and reactance per phase, (iii) total power, and (iv) total reactive voltamperes. **(10)**

b) Derive relation between line & phase voltage and line & phase current in three phase delta connected circuit. **(10)**

Q6 a) A 230/110 V, single-phase transformer takes an input of 350 VA at no load and at rated voltage. The core loss is 110 W. Find (i) no-load power factor, (ii) the iron loss component of no-load current, and (iii) magnetizing component of no-load current. **(10)**

b) Describe parts of DC machines and their use. **(10)**

Time: 2 hours **Maximum marks: 60**

NB:

1) Question No.1 is Compulsory

2) Attempt any Three questions from the remaining Five questions

3) Figures to the right indicate full marks

4) Atomic weight: Ca = 40, Mg = 24, S = 32, Cl = 35.5, C = 12, H = 1, O = 16, Na = 23.

Q.1 Attempt any **five** of the following: **(15)**

a) Compare temporary and permanent hardness (any 3 points).

b) At room temperature, H_2O is liquid and H_2S is a gas. Justify.

c) Comment on glass transition temperature (Tg).

d) Give difference between bonding and antibonding orbitals.

e) Identify how many phase/s are present in each of the following systems;

1) Mixture of N_2, H_2 and O_2
2) Mixture of rhombic and monoclinic Sulphur
3) Ethanol and water

f) Explain drawbacks of Kekule's Benzene structure.

g) Calculate COD of 25 ml of waste water sample in ppm which was refluxed with 10 ml of $K_2Cr_2O_7$ and after refluxing the excess unreacted dichromate required 6.5 ml of 0.1N FAS solution. A blank of 25 ml of distilled water on refluxing with 10 ml of $K_2Cr_2O_7$ solution required 27 ml of 0.1N FAS solution.

Q.2) a) Apply Gibb's phase rule to one-component (Water) system. **(6)**

b) Explain reverse osmosis with the help of principle, process, and diagram. Also give its advantages. **(5)**

c) Discuss the roles of plasticizer and lubricant in compounding of plastics. **(4)**

Q.3) a) Draw the Molecular Orbital diagram of CO molecule. Give its electronic configuration. Calculate its bond order and identify its magnetic behaviour. **(6)**

b) What is hydrogen bonding? Explain its types with examples. **(5)**

c) Calculate number average molecular weight of polymer which has 5 molecules of molecular weight of 10000, 3 molecules of molecular weight 30000 and 2 molecules of molecular weight 60000. **(4)**

Q.4) a) Explain Gibbs phase rule with its mathematical expression. Give its limitations and advantages. **(6)**

b) Comment on structure and bonding of pyrrole. **(5)**

c) Explain n-type and p-type conducting polymers with appropriate examples. **(4)**

Q.5) a) Define fabrication of plastics. Explain injection moulding process with neatly labelled diagram. Give its any two advantages. **(6)**

b) Distinguish between orbit and orbital (any three points). Explain the shapes of p-orbitals with their diagrams. **(5)**

c) Write equations representing EDTA titration. **(4)**
Calculate total hardness of water in ppm, if 50 ml of standard hard water sample containing 1 mg of pure $CaCO_3$ per ml consumed 20 ml of EDTA solution and 50 ml of hard water sample required 30 ml of EDTA solution.

Q.6) a) Explain the ion-exchange method for softening of water giving the following details: Principle, diagram, process, and Reactions. **(6)**

b) i) Define: Phase and Component. **(2)**

ii) An alloy of tin and lead contains 73% tin. Find the mass of eutectic in 1kg of solid alloy, if the eutectic contains 64% of tin. **(3)**

c) Draw Molecular Orbital diagram of O_2 molecule. **(2)**

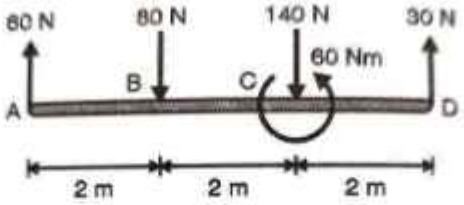
ii) Give synthesis of Kevlar. **(2)**

Time: 3 Hours **Max. Marks: 80**

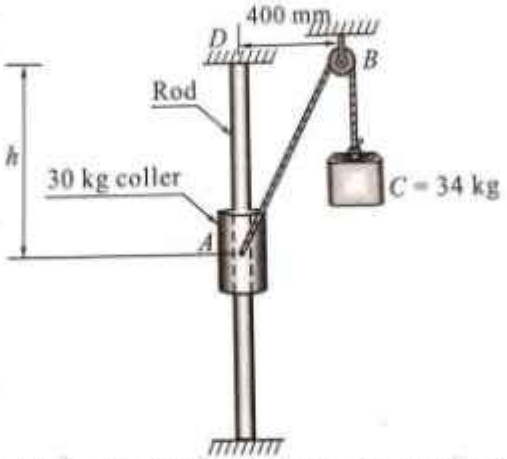
N. B. (1) Question No.1 is compulsory.
(2) Attempt any 3 questions from remaining five questions.
(3) Assume suitable data if necessary and mention the same clearly.
(4) Take g = 9.81 m/s², unless otherwise specified.

Q1. Solve any Four-

a) Replace the given force system in two parallel components at B & D. (05)



b) The 30 kg collar may slide on frictionless vertical rod and is connected to a 34 kg counterweight. Find the value of h for which the system is in equilibrium. (05)



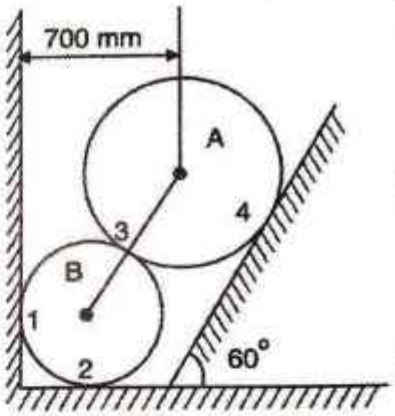
c) A block rests on a rough inclined plane making an angle of 30^0 with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, Find the mass of the block (in kg). (05)

d) A force acts at the origin in the direction defined by the angles $\theta_y = 65^0$ & $\theta_z = 40^0$. Knowing that the x-component of the force is 750 N. Determine (i) the other components. (ii) Magnitude of the force & (iii) the value of θx. (05)

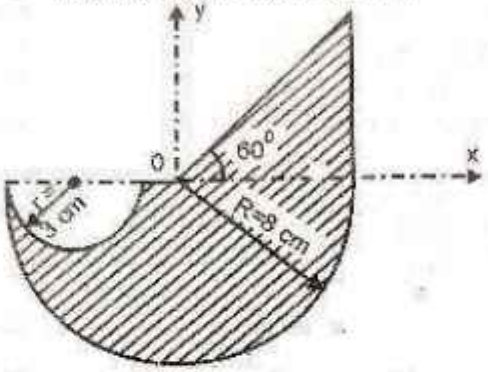
e) Explain the following with example- (05)
   i) General Plane motion.
   ii) Instantaneous centre of rotation.

f) If $x = 1 - t$ and $y = t^2$ where x and y are in meters and t is in second, determine the acceleration of the particle at t= 3 sec. (05)

Q 2 A) Two spheres A and B of weight 1000 N and 750 N respectively are kept as shown in figure. Determine the reactions at all contact points 1,2, 3 and 4. Radius of A is 400 mm and radius of B is 300 mm (07)



B) Find the centroid of the shaded areas shown in the fig. (07)

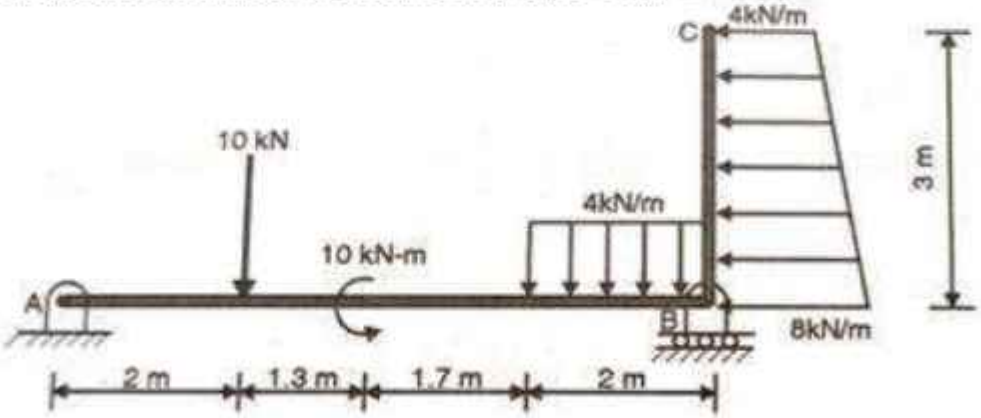


C) A particle starts from rest from origin and its acceleration is given by

$$a = \frac{k}{(x+4)^2}$$

Knowing that v = 4 m/s when x = 8 m. Find value of k, Also calculate the velocity when displacement is 10 m. (06)

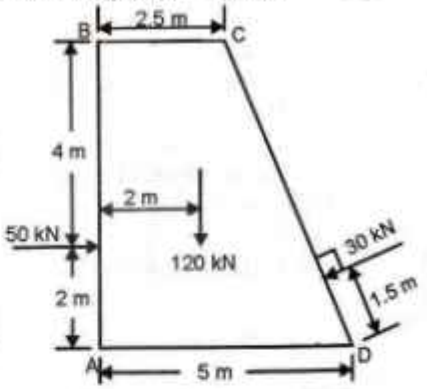
Q 3 A) Find the support reactions of beam loaded as shown in fig. (07)



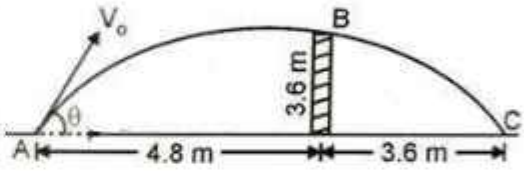
B) A uniform ladder rests with one end against a smooth vertical wall and other end on the ground, whose coefficient of friction is 0.5. If the inclination of ladder to the ground is 50^0, find the position of a man (whose weight is same as ladder) from the bottom of the ladder when the ladder is at the verge of slipping. The length of ladder is 8 m. (07)

C) A particle moves in x-y plane with acceleration components $a_x = -3m/s^2$ and $a_y = -16t\ m/s^2$. If its initial velocity is $V_0 = 50$ m/s directed at 35° to the x-axis, compute the radius of curvature of the path at t = 2 sec. Also calculate the velocity at t = 5 Sec. (06)

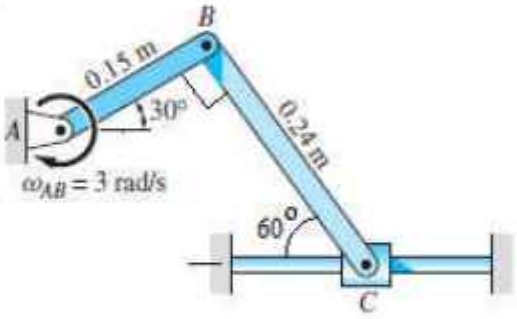
Q 4. A) A dam is subjected to three forces, 50 kN on the upstream face AB, 30 kN force on the downstream inclined face and its own weight of 120 kN as shown in figure. Determine the single force and locate its point of intersection with the base AD assuming all the forces to lie in a single plane. (07)



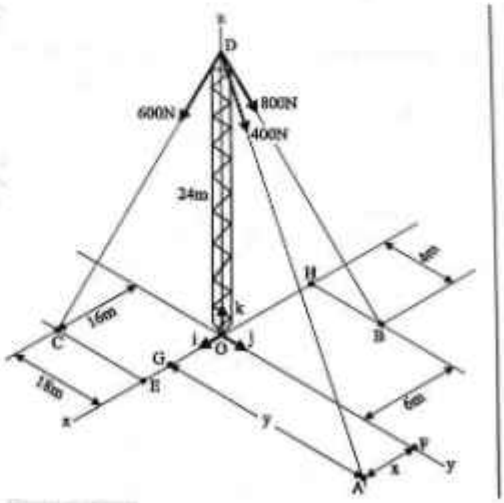
B) A boy throws a ball so that it may clear a wall 3.6 m high. The boy is at a distance of 4.8 m from the wall. The ball was found to hit the ground at a distance of 3.6 m on the other side of the wall. Find the least velocity with which the ball can be thrown, and the corresponding angle of projection. (07)



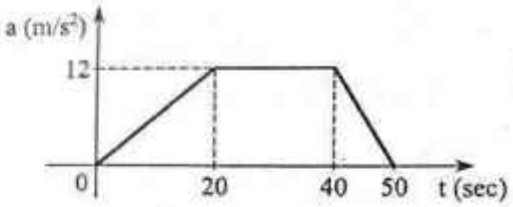
C) For the mechanism shown in figure, the angular velocity of bar *AB* is 3 rad/s clockwise. Using instantaneous center of rotation, calculate the angular velocity of bar *BC* and the velocity of slider *C* for this position (06)



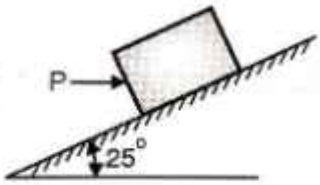
Q 5. A) A tower is held in position by three cables. Determine position (x, y) for the cable DA so that the resultant force exerted is directed from D to O. (10)



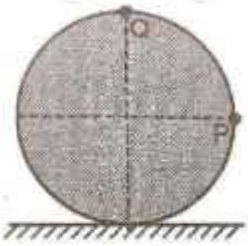
B) Fig shows a-t curve for a particle moving along x axis what is speed & distance covered by the particle after 50 sec? Find also the maximum speed & the time at which the speed is attain by the particle, also plot v-t & x-t curve. (10)



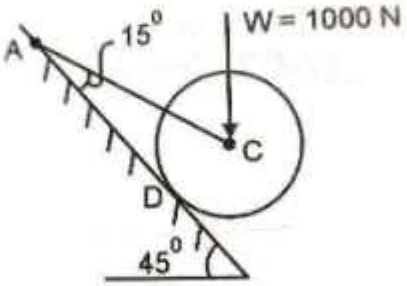
Q 6. A) A block of weight 800 N is acted upon by a horizontal force P as shown in figure, if $\mu_s = 0.35$ & $\mu k = 0.25$, determine the value of P for impending motion up the plane. (05)



B) A wheel of radius 0.75 m rolls without slipping on a horizontal surface. Determine the velocities of point P and Q shown in figure, when the velocity of centre of the wheel is 10 m/s towards right. (05)



C) 'A wheel of weight W = 1000 N rest on a smooth incline plane. It is kept from rolling down the plane by a string AC. Find the tension in the string and reaction at the point of contact D. (05)



D) A train starting from rest accelerates uniformly for 3 minutes, runs at constant speed for next 5 minutes and then come to rest in next 2 min. If it covers a total distance of 9 km, draw V- t diagram and find the acceleration and retardation in m/s² [05]

(Time: 2 Hours) [Total Marks: 60]

N.B.: (1) Question No. **1** is **compulsory.**
(2) Attempt **any three** questions from Q.2 to Q.6.
(3) Assume **suitable** data wherever **required.**
(4) Figures to the right indicate marks.

Q.1 Attempt any **FIVE** **(15)**

(a) The resistivity of cu is 1.72×10^{-8} ohm-m. Calculate the mobility of electron in Cu. Given that the number of electrons per unit volume is $10.41 \times 10^{28}/m3$

(b) Draw the following with reference to cubic unit cell:
(110),(011) and(101)

(c) Why do we see beautiful colors in thin film when it is exposed to sunlight?

(d) What are the properties of matter waves?

(e) Explain the principle & importance of super capacitors

(f) Explain the construction of LCD

(g) State de Broglie's hypothesis. Deduce an expression for the wavelength of de Broglie's matter waves.

Q.2 (a) What is thin film? Derive the conditions for maxima & minima due to interference of light reflected from thin film of uniform thickness **(08)**

(b) Derive Bragg's law. The Braggs angle corresponding to the first order reflection from (111) planes of a crystal is 30^0.wavelength of X-rays is1.75 A^0.Determine lattice constant of the crystal. **(07)**

Q.3 (a) Explain phase velocity & group velocity of a matter waves. Derive the one dimensional time dependent Schrodinger wave equation for matter waves. **(08)**

(b) What is photovoltaic effect? Explain the principle, working & applications of solar cell. **(07)**

Q.4 (a) Calculate electron and hole concentration in intrinsic silicon at room temperature if its electrical conductivity is 4×10^{-4} mho/m. (mobility of electron=0.14 m^2 /V-s & mobility of hole=0.04 m^2 /V-s) **(05)**

(b) Show that the energy of an electron in a box varies on the square of the natural numbers. **(05)**

(c) In Newton's ring experiment the diameter of 4^{th} and 12^{th} dark rings are 0.40cm and 0.70cm.find diameter of 20^{th} dark ring in reflected light. **(05)**

Q.5 (a) Write the applications of superconductors. **(05)**

(b) Show that Fermi energy level is placed in the center of the energy band gap in intrinsic semiconductors. **(05)**

(c) Explain Heisenberg's uncertainty principle with example & give its physical significance **(05)**

Q.6 (a) What is Meissner Effect? With the help of this effect show that superconductors are diamagnetic in Nature. **(05)**

(b) A parallel beam of light (wavelength-5870) is incident on a glass plate of Refractive Index is 1.5,such that the angle of refraction into the plate is 60^0.calculate the smallest thickness of the glass plate which will appear dark by reflection **(05)**

(c) If the uncertainty in position of an electron is $4x10^{-10}$ m, calculate the uncertainty in its momentum. **(05)**
