

Mech

Sub:-AM III (3 Hours)

[Total Marks : 100

- N.B. 1) Question No. 1 is compulsory.  
 2) Attempt any four questions out of the remaining six questions.  
 3) Figures to right indicate full marks.



Q 1. a) Find the Laplace transform of  $\frac{\cos 2t \cdot \sin t}{e^t}$  5

b) Prove that  $f(z) = x^2 - y^2 + 2ixy$  is analytic and find  $f'(z)$ . 5

c) Evaluate  $\int_0^{1+2i} z^2 dz$ , along the curve  $2x^2 = y$  5

d) Is the following matrix orthogonal? If not, can it be converted into an orthogonal matrix? If yes how.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix} \quad 5$$

Q 2. a) Find the orthogonal trajectory of the family of curves given by  $2x - x^3 + 3xy^2 = a$  6

b) Find Non-singular matrices P & Q such that PAQ is in normal form. Also find rank of A where A is

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \quad 6$$

c) Find the Laplace Transform of

i)  $e^{2t}(1+t)^2$       ii)  $\int_0^t u e^{-3u} \sin 4u du$  8

Q 3. a) Find the values of k for which the equations  $x + y + z = 1$ ,  $x + 2y + 3z = k$ ,  $x + 5y + 9z = k^2$  have solution. Solve them for these values of k. 6

b) Find the analytic function whose real part is  $u = x^4 - 6x^2y^2 + y^4$  6

c) Find inverse Laplace transform of following

i)  $\log\left(\frac{s+a}{s+b}\right)$       ii)  $\frac{s}{(s+3)(s-4)}$  8

Q 4. a) Find the image of  $|z-2|=3$ , under the transformation  $w=1/z$ . 6

b) Determine Eigen values and Eigen vectors for the matrix.  $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$  6

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- c) Evaluate i)  $\int_c \frac{z+2}{(z-3)(z-4)} dz$ , where  $c$  is the circle  $|z|=1$   
 ii)  $\int_c \frac{dz}{z+3}$ , where  $c$  is the circle  $|z|=2$

Q 5. a) Express the following matrix as the sum of symmetric and skew-symmetric matrix

$$A = \begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix}$$



b) Evaluate  $\int_0^{\infty} \frac{e^{-t} - e^{-3t}}{t} dt$

c) Obtain Taylor's and Laurent's expansions of  $f(z) = \frac{z-1}{z^2-2z-3}$  indicating ROC

- Q 6. a) Find the bilinear transformation, which maps  $z = \infty, i, 0$  on to the points  $w = 0, i, \infty$ .  
 b) Evaluate the following using Residues theorem

$$\int_c \frac{z-1}{(z+1)^2(z-2)} dz \text{ where } c \text{ is the circle } |z-i| = \frac{1}{2}$$

c) Find  $L^{-1} \left\{ \frac{s^2}{(s^2+a^2)^2} \right\}$  using convolution theorem.

Q 7. a) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$

b) Verify that  $A(\text{adj } A) = |A|I$  for  $A = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 1 & 1 \\ 4 & -5 & 2 \end{bmatrix}$

c) Using Laplace Transform solve the following differential equation with given conditions

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0, \text{ where } y(0) = 0, y'(0) = 4$$



SE sem - III  
Subj: - Thermodynamics  
Mech

May 15  
(3 Hrs)

May-15  
CBUS 20-5-15  
QP Code : 4775

[Total Marks : 80]

- Question No 1 is compulsory.
- Answer any three out of remaining five questions.
- Assumptions made should be clearly stated .
- Assume suitable data wherever required, but justify the same.
- Use of Mollier Chart, Steam Table, permitted.



1 Answer any Four of the following :

[20]

- Prove that energy is a property of the system
- Explain how heat pump is more efficient for heating application than electrical heating.
- Determine the maximum work obtainable from a Heat Engine exchanging heat with two finite bodies of equal heat capacities at temperatures  $T_1$  and  $T_2$  ( $T_1 > T_2$ )
- What is cut off ratio. Discuss its effect on the thermal efficiency of Diesel Engine.
- Define Joule Thompson Coefficient and explain its significance.
- Write a note on Adiabatic Flame Temperature.

2 a) State and explain the equivalence of Kelvin Planck and Clausius statements of Second Law of Thermodynamics. [6]

b) A reciprocating air compressor takes in  $2 \text{ m}^3/\text{min}$  at  $0.11 \text{ MPa}$ ,  $20^\circ \text{C}$ , which it delivers at  $1.5 \text{ MPa}$ ,  $111^\circ \text{C}$  to an aftercooler where the air is cooled at constant pressure to  $25^\circ \text{C}$ . The power absorbed by the compressor is  $4.15 \text{ kW}$ . Determine the heat transfer in the compressor and the aftercooler. [10]

c) Define COP of Heat Pump and refrigerator and derive the relationship between the two. [4]

Q 3 a) Two kg of air at  $500 \text{ kPa}$ ,  $80^\circ \text{C}$  expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at  $100 \text{ kPa}$ ,  $5^\circ \text{C}$ . For this process determine (i) the maximum work (ii) the available energy (iii) the irreversibility. [10]

b) A heat engine is used to drive a heat pump. The heat transfers from the heat engine and from heat pump are used to heat the water circulating through the radiators of the building. The efficiency of the heat engine is  $27\%$  and the COP of the pump is  $4$ . Evaluate the ratio of heat transfer to the circulating water to the heat transfer to the heat engine. [10]

4 a) Plot the Rankine cycle on T-S diagram and derive an expression for thermal efficiency of the cycle. List different methods of improving the performance of the cycle. Discuss any one method in brief. [8]

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- b) Water at  $40^\circ\text{C}$  is continuously sprayed into a pipeline carrying 5 tons of steam per hour at 5 bar,  $300^\circ\text{C}$ . At a section downstream where the pressure is 3 bar, the quality is to be 95%. Find the rate of water spray in kg/hr.
- c) Derive an expression for ratio of Heat Capacities ( $\gamma$ ) in terms of isothermal compressibility ( $k$ ) and adiabatic compressibility ( $k_s$ ).
- 5 a) A cylinder contains  $0.084\text{ m}^3$  of hydrogen at 1.05 bar and  $18^\circ\text{C}$ . It is compressed adiabatically to 14 bar and then expanded isothermally to the original volume. The characteristic constant for hydrogen is  $4200\text{ kJ/kg K}$  and its specific heat at constant pressure is  $14.29\text{ kJ/kg K}$ . Determine the final pressure of the gas and the amount of heat added during isothermal expansion. Also find the amount of heat which must be extracted from the gas to reduce it to initial state of pressure.
- b) An engine working on dual cycle uses a compression of 14. The intake pressure and temperature are 1 bar and 330 K. The explosion ratio is 1.4. The heat supplied during constant pressure process is twice that at the constant volume process. Determine per kg of air (i) the percentage cut-off ratio (ii) work done and (iii) air standard efficiency.
- 6 a) An engine uses n butane ( $\text{C}_4\text{H}_{10}$ ) as liquid fuel. It is supplied with 40% excess air. Both fuel and air enter at 1 atmosphere pressure and 298 K. The products of combustion leave at 600 K. Heat lost to the surroundings is 30% of power. The engine develops 60 kW of power. Determine the mass flow rate of fuel in kg/hr. The following data is applicable :

Substance	$h_f^0$ (kJ/kgmole)	$h_{298\text{K}}$ (kJ/kgmole)	$h_{600\text{K}}$ (kJ/kgmole)
$\text{C}_4\text{H}_{10}$ (l)	-125150	0	-
$\text{O}_2$ (g)	0	8624	18260
$\text{N}_2$ (g)	0	8660	17569
$\text{CO}_2$ (g)	-241830	8769	22285
$\text{H}_2\text{O}$ (g)	-393520	9856	20402

- b) In a single heater regenerative cycle the steam enters the turbine at 30 bar  $400^\circ\text{C}$  and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Determine :
- The efficiency and steam rate of cycle
  - The increase in efficiency and steam rate as compared to Rankine Cycle without regeneration.



- N. B. :** (1) Question No.1 is **compulsory**.  
 (2) Answer any **three** questions from remaining **five** questions.  
 (3) Assume suitable data wherever required but justify the same.  
 (4) Answer to the questions showed be grouped and written together.

1. Answer any **four** questions:-

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- (a) Draw stress strain curve for ductile material & explain salient point on it.  
 (b) What is sagging and hogging moment in bending moments gives its sign conventions.  
 (c) Derive flexural formula

$$\frac{M}{I} = \frac{F}{Y} = \frac{E}{R} \quad \text{OR} \quad \frac{M}{I} = \frac{6}{Y} = \frac{E}{R}$$

State assumption made in simple bending.

- (d) What are the assumption made in theory of pure torsion & derive equation of Torsion.  
 (e) What are the assumptions made in the analysis of struts & column by Euler's buckling theory? What are its limitations?

2. (a) A copper rod 36 mm diameter is encased and rigidly attached at the end of a steel tube which is 50 mm external diameter, thickness of metal being 5mm. The composite section is subjected to an axial pull of 100 KN. Find the stress induced in each metal & the extension on the length of 1.5 m. Take  $E_s = 2 \times 10^5 \text{ N/mm}^2$  and  $E_c = 1.1 \times 10^5 \text{ N/mm}^2$ .  
 (b) Determine the value of load W, if support reactions are equal. Also draw shear force diagram and bending moment diagram for the beam loaded as shown in fig.1.

10

10

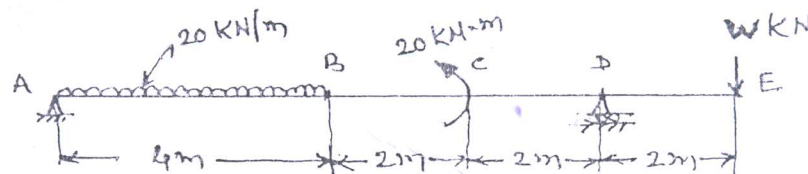


Fig. 1

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3. (a) A cast iron beam section is of I section with top flange 80mm x 20mm thick, bottom flange 160 mm x 40 mm thick and the web 200mm deep and 20 mm thick. The beam is freely supported on a span of 5 meter. If the tensile stress is not to exceed 20 N/mm<sup>2</sup>, find the safe uniformly distributed load, which the beam can carry, also find the maximum compressive stress. 10
- (b) A solid shaft is required to transmit 330KW at 120 rpm. The shear stress of material must not exceed 80 N/mm<sup>2</sup>, find the diameter of shaft required, if the above shaft is to be replaced by hollow one with diameter ratio 3:5 and maximum shear stress remain unchanged. Calculate the percentage saving in weight that could be obtained. 10

4. (a) Show that the strain energy equation in a simply supported beam carrying a UDL w/m is given by, 10

$$U = \frac{W^2 L^5}{240EI}$$

- (b) Determine slope at the supports and deflection under other points. Also find maximum slope and its position for fig. 2 Take EI = constant. 10

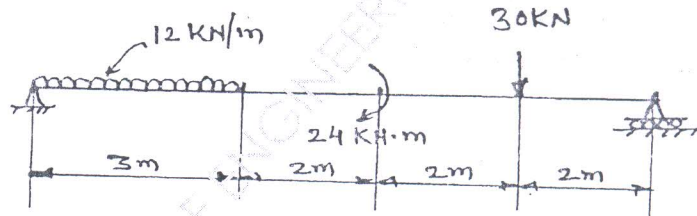
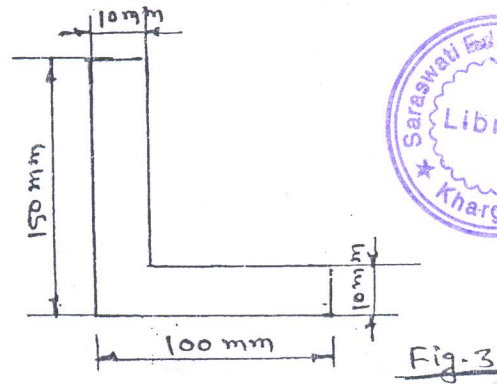


Fig. 2.

5. (a) A cylindrical vessel with hemispherical ends is 1m long on its cylindrical portion and has 0.5 m diameter. Thickness of wall on cylindrical portion is 6 mm. Taking internal pressure as 1MPa,  $E = 200\text{GPa}$  &  $\mu = 1/m = 0.3$ . 10
- Determine
1. Thickness of wall of hemispherical portion
  2. Change in volume of vessel
- (b) A hollow column of C.I. whose outside diameter is 200 mm and has a thickness of 20mm, it is a 4.5 m long and is fixed at the both ends. calculate the safe load by Rankin's formula using a factor of safety of 4. Calculate slenderness ratio and the ratio of Euler's and Rankin's critical load. Take  $6c = 550\text{ N/mm}^2$ ,  $a = 1/1600$  and  $E = 8 \times 10^4\text{ N/mm}^2$  10

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6. (a) An area in the form of L section is shown in the fig. 3 (i) find the moment of inertia  $I_{xx}$ ,  $I_{yy}$  and  $I_{xy}$  about its centroidal axes. (ii) Also determine the principal moment of inertia. 10



- (b) A steel bar is placed between two copper bars each having the same area and length, as the steel bar at  $15^\circ$ . At this stage they are rigidly connected together at both the ends. When the temperature is raised to  $315^\circ$ , the length of the bars increases to 1.5 mm. Determine the original length and the final stresses in the bars. 10
- Take  $E_s = 2.1 \times 10^5 \text{ N/mm}^2$ ,  $E_c = 1 \times 10^5 \text{ N/mm}^2$ ,  $\alpha_s = 0.000012 \text{ per } ^\circ\text{C}$ ,  $\alpha_c = 0.0000175 \text{ per } ^\circ\text{C}$ .

JP-Con.: 10330-15.



N.B. : (1) Question No. 1 is **compulsory**.

(2) Attempt **any three** questions out of **remaining five** questions.

1 (a) Find the Laplace transform of  $te^{-t} \cosh 2t$  5

(b) Find the fixed points of  $w = \frac{3z-4}{z-1}$ . Also express it in the normal form 5

$\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + \lambda$  where  $\lambda$  is a constant and  $\alpha$  is the fixed point. Is this transformation parabolic?

(c) Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along the path i)  $y=x$ , ii)  $y=x^2$  5

(d) Prove that  $f_1(x)=1$ ,  $f_2(x)=x$ ,  $f_3(x) = \frac{3x^2-1}{2}$  are orthogonal over  $(-1,1)$  5

2. (a) Find inverse Laplace transform of  $\frac{2s}{s^4+4}$  6

(b) Find the image of the triangular region whose vertices are  $i$ ,  $1+i$ ,  $1-i$  under the transformation  $w = z + 4 - 2i$ . Draw the sketch. 6

(c) Obtain fourier expansion of  $f(x) = |\cos x|$  in  $(-\pi, \pi)$ . 8

3. (a) Obtain complex form of fourier series for  $f(x) = \cosh 2x + \sinh 2x$  in  $(-2,2)$ . 6

(b) Using Carrk-Nicholson simplified formula solve  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$  given 6

$u(0,t)=0, u(4,t)=0, u(x,0) = \frac{x}{3} (16-x^2)$  find  $u_{ij}$  for  $i=0,1,2,3,4$  and  $j=0,1,2$

(c) Solve the equation  $y + \int_0^t y dt = 1 - e^{-t}$  8



4. (a) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$
- (b) Find half - range cosine series for  $f(x)=e^x, 0<x<1$
- (c) Obtain two distinct Laurent's series for  $f(z) = \frac{2z-3}{z^2-4z-3}$  in powers of  $(z-4)$  indicating the regions of convergence.
5. (a) Solve  $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$  by Bender - Schmidt method, given  $u(0, t) = 0$ ,  $u(4, t) = 0$ ,  $u(x, 0) = x(4-x)$ . Assume  $h=1$  and find the values of  $u$  upto  $t = 5$
- (b) Find the Laplace transform of  $e^{-4t} \int_0^t u \sin 3u du$
- (c) Evaluate  $\int_C \frac{z+3}{z^2+2z+5} dz$  where  $C$  is the circle i)  $|z| = 1$ , ii)  $|z+1-i|=2$
6. (a) Find inverse Laplace transform of  $\frac{s}{(s^2-a^2)^2}$  by using convolution theorem.
- (b) Find an analytic function  $f(z) = u+iv$  where  $u+v=e^x (\cos y + \sin y)$
- (c) Solve the equation  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$  for the conduction of heat along a rod of length  $l$  subject to following conditions
- (i)  $u$  is not infinity for  $t \rightarrow \infty$
- (ii)  $\frac{\partial u}{\partial x} = 0$  for  $x=0$  and  $x=l$  for any time  $t$
- (iii)  $u=lx-x^2$  for  $t=0$  between  $x = 0$  and  $x=l$

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(OLD COURSE)  
(3 Hours)QP Code : 4533  
[Total Marks : 100]

- Note : 1. Question No.1 is compulsory.  
2. Answer any four from the remaining six questions.

1. a) If  $L\{f(t)\} = \frac{s}{s^2 + s + 4}$ , find  $L\{e^{-2t} f(2t)\}$

[5]

b) Find the orthogonal trajectory of the family of curves given by  $2x - x^3 + 3xy^2 = a$ .

[5]

c) Evaluate  $\oint_C \log z \, dz$  where C is  $|z|=1$

[5]

d) Express the matrix  $A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$  as  $P + iQ$  where P is real Skew-symmetric matrix and Q is real Symmetric matrix.

[5]

2. a) Determine the analytic function whose imaginary part is  $(x^4 - 6x^2y^2 + y^4) + (x^2 - y^2) + 2xy$

[6]

b) Evaluate  $\int_C \frac{4z-1}{z^2-3z-4} dz$  where C is the ellipse  $x^2 + 4y^2 = 4$ .

[6]

c) Reduce to normal form and find rank of the following matrix :

[8]

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

3. a) Solve the Differential Equations using Laplace Transformation

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t} \quad y(0) = -3, \quad y'(0) = 5$$

[6]

b) Find the sum of the residue at singular points of  $f(z) = \frac{z}{(z-1)^2(z^2-1)}$

[6]

c) If  $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ , then prove that  $3 \tan A = A \tan 3$

[8]

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4. a) Prove that  $u = \log \sqrt{x^2 + y^2}$  is harmonic and find its harmonic conjugate
- b) Examine whether the vectors  $X_1 = [3 \ 1 \ 1]$   $X_2 = [2 \ 0 \ -1]$   $X_3 = [4 \ 2 \ 1]$  are linearly independent or dependent.
- c) Find Inverse Laplace Transform of (i)  $\cot^{-1}(as)$
- (ii)  $\frac{(s+3)^2}{(s^2+6s+5)^2}$  using convolution theorem
5. a) Find the image of the rectangle bounded by  $x=0, y=0, x=1, y=2$  under the transformation  $w = (1+i)z + (2-i)$ . Sketch the region.
- b) Find the Eigen value and Eigen vector of  $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$
- c) Evaluate the Integral  $\int_0^{\infty} \frac{e^{-\sqrt{2}t} \sin t \sinh t}{t} dt$
6. a) Evaluate  $\int_0^{2\pi} \frac{d\theta}{25-16\cos^2\theta}$
- b) Verify Cayley-Hamilton Theorem for  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and hence find  $A^{-1}, A^3 - 5A^2$ .
- c) Find Laplace Transform of (i)  $\int_t^{\infty} \frac{\cos u}{u} du$
- (ii)  $t \left( \frac{\sin t}{e^t} \right)^2$
7. a) Find Laplace Transform of the following periodic functions :
- $f(t) = K \frac{t}{T}$  for  $0 \leq t \leq T$  ,  $f(t+T) = f(t)$
- b) Find the Bilinear Transformation that maps the points  $z = 1, i, -1$  into  $w = i, 0, -i$ .
- c) Obtain Laurent and Taylor's series for  $\frac{z-1}{z^2-2z-3}$  indicating region of convergence