(3 Hours) **Total Marks : 80**

N.B. : (1) Question Number 1 is compulsory
(2) Solve any three questions from the remaining questions
(3) Make suitable assumptions if needed
(4) Assume appropriate data whenever required. State all assumptions clearly.

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1\. a. Define the following with suitable example **5**
a)Ring b) Cyclic Group c) Monoid d)Normal Subgroup e) Planar Graph

b. Check whether $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is a tautology **5**

c. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2,3 or 5. **5**

d. Prove by mathematical induction that **5**
$2+5+8+\ldots\ldots+(3n-1) = n(3n+1)/2$

2 a Define Equivalence Relation. Let A be a set of integers, Let R be a Relation on AXA defined by (a,b) R (c,d) if and only if ad = bc. Prove that R is an Equivalence Relation **8**

b. Let A={a, b, c, d, e} **8**

$$M_R = \begin{matrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

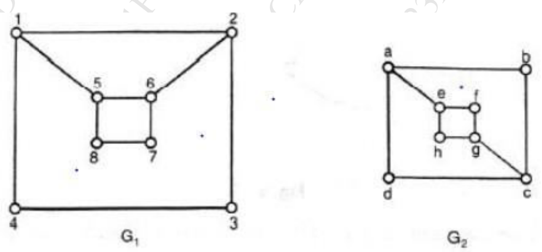
Find the transitive closure of it using Warshall's algorithm.

c Let G be a group. Prove that the identity element 'e' is unique. , **4**

3 a Prove that set G = {1,2,3,4,5,6} is a finite abelian group of order 6 with respect to multiplication module 7 **8**

b Give the exponential generating function for the sequences **8**
i) {1,1,1…}
ii) {0,1,0,-1,0,1,0,-1…}.

c Determine whether the following graphs are isomorphic. Justify your answer. **4**

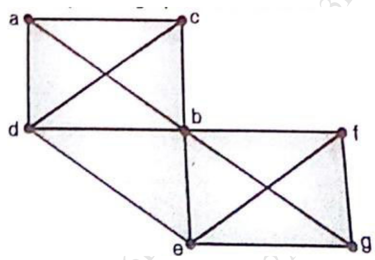


4\. a A Function $f: R - \left\{\frac{7}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$ is defined as $f(x) = (4x - 5)/(3x - 7)$ **8**

Prove that f is Bijective and find the rule for f^{-1}

b Show that (2,5) encoding function $\mathbf{e:B^2 \rightarrow B^5}$ defined by **8**
e(00)=00000
e(01)=01110
e(10)=10101
e(11)=11011
is a group code.

c Check whether Euler cycle and Euler Path exist in the Graph given below. If yes Mention them **4**



5\. a Consider the Set A={1,2,3,4,5,6} under multiplication Modulo 7. **8**
1) Prove that it is a Cyclic group.

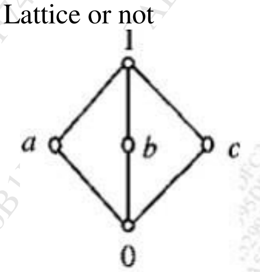
2) Find the orders and the Subgroups generated by {2,3}and {3,4}

b State and explain the extended Pigeonhole principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. **8**

c Functions f,g,h are defined on a set X={a,b,c} as **4**
f={(a,b),(b,c),(c,a)}
g={(a,b),(b,a),(b.b)}
h={(a,a),(b,b),(c,a)}
i) Find fog, gof . Are they equal?
ii) Find fogoh and fohog?

6\. a Draw the Hasse Diagram of $\mathbf{D_{72}}$ and $\mathbf{D_{105}}$ and check whether they are Lattice. **8**

b Define Bounded Lattice and Distributive Lattice. Check if the following diagram is a Distributive Lattice or not **8**



c Define the following with suitable example. **4**
a)Hamiltonian path b) Euler Circuit c) Sub Lattice d)Group e) Surjective Function

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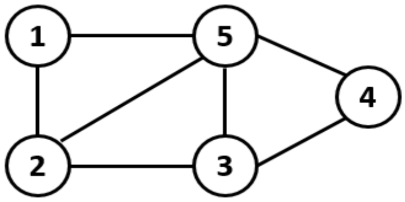
(3 Hours) **Total Marks: 80**

N.B: (1) Question No. 1 is compulsory
(2) Attempt any three questions out of the remaining five questions
(3) Figures to the right indicate full marks
(4) Make suitable assumptions wherever necessary with proper justifications

Q1 A What is Hashing? Explain Hash collision with example. [05]
B Explain types of Double Ended Queue with example. [05]
C Differentiate between arrays and linked list. [05]
D List different data structures along with one application. [05]

Q2 A Construct Binary Search Tree by inserting the following elements in sequence 45, 28, 34, 63, 87, 76, 31, 11, 50, 17. [10]
B Write a program in C to implement Queue using singly linked list. [10]

Q3 A Write a program to perform the following operations on doubly linked list: [10]
i) Insert a node at the front of the list
ii) Delete a node from the front of the list
iii) Count the number of nodes in the list
iv) Display the list
B Define Graph. Show the adjacency matrix and adjacency list representation for the following graph [05]



C Explain stack overflow and underflow conditions with example. [05]

Q4 A Write an algorithm to check the well-formedness of parenthesis. [10]
B Show the result of inserting the elements 16, 18, 5, 19, 11, 10, 13, 21, 8, 14 one at a time into an initially empty AVL tree. [10]

Q5 A Define tree traversal. Explain binary tree traversal techniques with example. [10]
B A hash table of size 10 uses linear probing to resolve collisions. The key values are integers and the hash function used is key%10. Draw the table that results after inserting in the given order the following values: 28, 55, 71, 38, 67, 11, 10, 90, 44, 9 [10]

Q6 A Explain Depth First Search and Breadth First Search traversal of a graph with example. [10]
B Construct Huffman tree and determine the code for each symbol in the string “PROGRAMMING”. [10]

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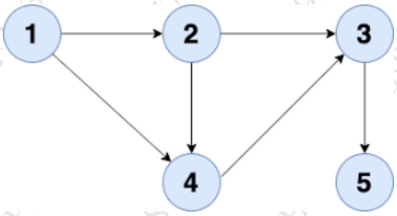
(3 Hours) **Total Marks: 80**

N.B: (1) Question No. 1 is compulsory
(2) Attempt any three questions out of the remaining five questions

Q.1	(a)	Explain various types of data structures with example.	**5**
	(b)	Define Graph and explainvarious graph representationtechniques.	**5**
	I	Convert the following expression to postfix. (f-g) * ((a+b) * (c-d))/e	**5**
	(d)	Differentiate between B tree and B+ tree.	**5**
Q.2	(a)	Apply linear probing and quadratic probing hash functions to insert values in the Hash table of size 10. Show number of collisions occurs in each technique. 27, 72, 63, 42, 36, 18, 29,101	**10**
	(b)	Construct B+ tree of order 3 for the following dataset 90, 27, 7, 9, 18, 21, 3, 4, 16, 11, 1, 72	**10**
Q.3	(a)	Write BFS algorithm. Show BFS traversal for the following graph with all the steps.  	**10**
	(b)	Write a C program to implement linear queue using array.	**10**
Q.4	(a)	Write a program to perform the following operations on the Singly linked list: i. Insert a node at the end ii. Delete a node from the beginning iii. Search for a given element in the list iv. Display the list	**10**
	(b)	Write a C program to implement Stack using Linked List	**10**
Q.5	(a)	Write a program to evaluate postfix expression using stack data structure	**10**
	(b)	Construct AVL for following elements 50, 25, 10, 5, 7, 3, 30, 20, 8, 15	**10**
Q.6	(a)	Construct Binary Tree from following traversal. In–order Traversal: D B H E I A F J C G Post order Traversal: D H I E B J F G C A	**10**
	(b)	Write a C program for polynomial addition using a Linked-list.	**10**

Time:3Hours **Max. Marks: 80**

N.B.
1) **Q.1 is compulsory.**
2) Solve any 3 questions out of remaining 5 questions.
3) Assumptions made should be clearly stated.
4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

a) Check if $(p\rightarrow q)\rightarrow[(\sim p\rightarrow q)\rightarrow q]$ is a tautology? **5**

b) Draw the Hasse diagram for [{2,4,5,8,10,12,20,25}, /]. Is it a Poset? **5**

c) Define Eulerian and Hamiltonian Graph. Give examples of following type of graph **5**
 i) Eulerian but not Hamiltonian
 ii) Not Eulerian but Hamiltonan

d) Explain types of Quantifiers . Represent the following sentences using Quantifiers **5**
 i) All hardworking students are clever.
 ii) There is a student who can speak Hindi but does not know Marathi

e) State the Pigeonhole principle and prove that in any set of 29 persons at least five persons must have been born on the same day of the week **5**

Q.2

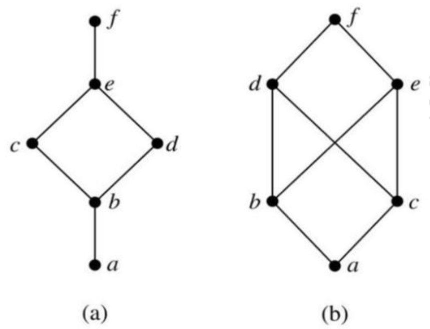
a) Show that the set of all positive rational numbers forms an abelian group under the composition * defined by a*b=(ab)/2 **10**

a) What is a transitive closure? Explain Warshall's algorithm for finding transitive closure with an example. **10**

Q.3

a) By using mathematical induction, prove that the given equation is true for all positive integers. **6**
$1 \times 2 + 3 \times 4 + 5 \times 6 + \dots + (2n - 1) \times 2n = n(n+1)(4n-1)/3$

b) Define Lattice? Which of the following is lattice? **8**



c) Determine the sequence of which recurrence relation is $a_n = 2a_{n-1} - a_{n-2}$ with initial conditions $a_1 = 1.5$, $a_2 = 3$. **6**

Q.4

a) Let A={1, 3,6,9,15,18,21} & R be the relation of divisibility. **8**

i) Write the pairs in a relation set R.

ii) Construct the Hasse diagram.

iii) What are the Maximal and Minimal elements?

iv) Is this poset a distributive lattice? Justify your answer.

b) **6**

Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine (3, 6) group code $e_H : B^3 \rightarrow B^6$

c) Write a short note on Types of Graphs. **6**

Q.5

a) Let(Z,*) be an algebraic structure, where Z is set of integers and the operation * is defined by a*b= maximum of (a,b). Is (Z,*) a Semigroup? Is (z,*) a Monoid? Justify your answer. **8**

b) Define the term Surjective function. Let E be the set of all even numbers then f: N-->E:f(x)=2x , check if it is surjective,bijective?Justify your answer. **4**

c) Give the examples of relation R on S = {a, b, c,d} having stated property. **8**

i) R is an equivalence relation.

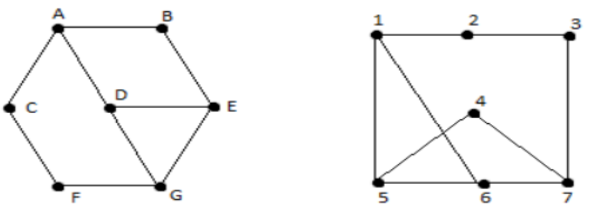
ii) R is symmetric but not transitive

iii) R is both symmetric and antisymmetric

iv) R is neither symmetric nor antisymmetric.

Q.6

a) Define Isomorphic graphs and check whether the following graphs are Isomorphic ? **8**



b)In a group(G,*) , Prove that the inverse of any element is unique and identity element is also unique. **6**

c) Define Relation. Let **6**

f: R→R is defined as $f(x) = x^2$

g: R→R is defined as $g(x) = 3x^2+1$

h: R→R is defined as $h(x) = 9x-2$

find (hof)og, go(foh).

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(Time: 3 hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks

Q.1 a) Find $L(t + e^t + \cos t)^2$ [5]

Q.1 b) Find the Fourier series for $f(x) = x\, sinx$ in $(-\pi, \pi)$ [5]

Q.1 c) Find Karl Pearson's coefficients of correlation between X and Y from the following data [5]

X	100	200	300	400	500
Y	30	40	50	60	70

Q.1 d) If $f(z) = (x^3 + axy^2 + bxy) + i(3x^2y + cx^2 + y^2 + dy^3)$ is analytic, then find a, b, c, d [5]

Q.2 a) A random variable X has the following probability function [6]

X	1	2	3	4	5	6	7
P(X=x)	k	2k	3k	k^2	k^2+k	$2k^2$	$4k^2$

Find i) k, ii) P(X≥ 4), iii) P(X < 5)

Q.2 b) Determine the analytic function whose real part is $u = e^x$cosy [6]

Q.2 c) Evaluate $\int_0^\infty e^{-t} \cosh t\, \cos 2t\, dt$. [8]

Q.3 a) Obtain the Fourier series for $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval (0, 2π) [6]

Q.3 b) A continuous random variable X has the p.d.f. $f(x) = kx^2e^{-x},\ x \geq 0$ [6]

Find i) k, ii) $P(1 \leq x \leq 2)$

Q.3 c) Find $L^{-1}\left[\frac{s+29}{(s+4)(s^2+9)}\right]$ using partial fraction method [8]

Q.4 a) Find $L[f(t)],\ where\ f(t) = \cos t, 0 < t < \pi\ \ and\ f(t) = 0,\ t > \pi$ [6]

Q.4 b) Compute Spearman's rank correlation coefficient for the following data [6]

X	18	20	34	52	12
Y	39	23	35	18	46

Q.4 c) Obtain the Fourier series for [8]

$$f(x) = \begin{cases} 1, & 0 \le x \le \pi \\ 2 - \frac{\pi}{x}, & \pi \le x \le 2\pi \end{cases}$$

Q.5 a) Find $L^{-1}\left[\frac{4s+13}{s^2+8s+13}\right]$ [6]

Q.5 b) Find $L[(1+sin2t)^2]$ [6]

Q.5 c) Find the line of regression of Y on X for the following data [8]

X	5	6	7	8	9	10	11
Y	11	14	14	15	12	17	16

Q.6 a) Find mean and variance for the following distribution [6]

X	8	12	16	20	24
P(X = x)	1/8	1/6	3/8	1/4	1/12

Q.6 b) Find $i)\ L^{-1}[cot^{-1}2s]\ \ ii)\ L^{-1}\left[\log\left(1+\frac{4}{s^2}\right)\right]$ [6]

Q.6 c) Prove that the function $f(z) = e^{2z}$ is analytic. Also, find its derivative. [8]

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(Time: 3 hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $t\sqrt{1+\sin t}$ [5]

(b) Find the constants a, b ,c, d, e if [5]

f(z) = $(ax^3 + b\,x\,y^2 + 3\,x^2 + c\,y^2 + x) + i\,(d\,x^2\,y - 2\,y^3 + e\,x\,y + y)$ is analytic.

(c) Calculate the Spearman's rank correlation coefficient R [5]

X :	85 ,	74 ,	85 ,	50,	65,	78,	74,	60,	74,	90
Y :	78	91,	78,	58,	60,	72,	80,	55,	68,	70

(d) Find inverse Laplace transform of $tan^{-1}\left(\frac{s+a}{b}\right)$. [5]

Q2.

(a) Find the Laplace transform of $e^{-4t}\int_0^t u\sin 3u\,du$ [6]

(b) find the value of k if the function f (x) = k x^2 ($1 - x^3$) , $0 \le x \le 1.$

F (x) = 0 otherwise

Is a probability density function. find mean and variance. [6]

(c) Obtain the Fourier series to represent f (x) = x^2 in (0, 2π)

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$ .... [8]

Q3.

(a) Find the analytic function f (z) = u + iv such that [6]

$u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$.

(b) Using convolution theorem Find inverse Laplace transform of $\frac{s^2}{(s^2+9)(s^2+4)}$. [6]

(c) Fit a second-degree parabolic curve to the following data

Year (x)	: 1974	1975	1976	1977	1978	1979	1980	1981
Production (y)	: 12	14	26	42	40	50	52	53.

[8]

Q4.

(a) Obtain the Fourier series to represent f (x) = 9 - x^2 in (-3, 3). [6]

(b) . Find the coefficients of regression and hence obtain the equation of the lines of Regression for the following data

X :	78 ,	36 ,	98 ,	25 ,	75,	82,	90,	62,	65,	39 .
Y :	84 ,	51,	91,	60,	68,	62 ,	86,	58,	53,	47.

[6]

(c) Prove that $\int_0^\infty e^{-t} \frac{\sin 2t+\sin 3t}{t}$ dt = $\frac{3\pi}{4}$. [8]

Q5.

(a) Find the orthogonal trajectories of the family of curves 3 x^2y + 2x^2 - y^3- 2 y^2 = c. [6]

(b) If X denotes the outcome when a fair die is tossed, find Moment generating function Of X and hence find the mean and variance of X. [6]

(c) Obtain the half range cosine series of f (x) = x ($\pi - x$) in (0, π)

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4}$ …….. [8]

Q6.(a) Find inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. [6]

(b) The probability density function of a random variable X is

X	: 0	1	2	3	4	5	6
P (X =x)	: k	3k	5k	7k	9k	11k	13k

Find k , p (X < 4) , P (3 < X ≤ 6) . [6]

(c) Verify Laplace equation for u = $\left(r + \frac{a^2}{r}\right)\cos\theta$.also find v and f (z) . [8]

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