



**University of Mumbai**

**Examination 2021 under cluster \_\_ (Lead College: \_\_)**

**Examinations Commencing from 15<sup>th</sup> June 2021 to 24<sup>th</sup> June 2021**

**Program: BE (Information Technology)**

**Curriculum Scheme: Rev 2016 (CBCGS)**

**Examination: SE Semester III**

**Course Code: ITC301 and Course Name: Applied Mathematics III**

**Time: 2-hours**

**Max. Marks: 80**

<b>Q1.</b>	<b>Choose the correct option for following questions. All the Questions are compulsory and carry equal marks</b>
1.	$I = \int_0^{\infty} e^{-t} \frac{\sin t}{t} dt$ then value of I is
Option A:	$\pi/2$
Option B:	$\pi/4$
Option C:	$-\pi/4$
Option D:	$\pi$
2.	On set of integers , a relation R is defined as $aRb$ iff $a \leq b$ then which of the following is true ?
Option A:	R is equivalence
Option B:	R is symmetric
Option C:	R is not transitive
Option D:	R is reflexive
3.	$f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = 2x + 1$ for $x \in \mathbb{R}$ . Find rule for $f^{-1}(x)$
Option A:	$f^{-1}(x) = \frac{x+1}{2}$
Option B:	$f^{-1}(x) = \frac{x-1}{2}$
Option C:	$f^{-1}(x) = 2x - 1$
Option D:	$f^{-1}$ doesn't exist
4.	Inverse Laplace transform of $\frac{1}{s^2-2s+1}$ is
Option A:	$e^t$
Option B:	$te^t$
Option C:	$\sin t$
Option D:	$te^{-t}$
5.	$S = [0,1]$ then S is
Option A:	countable set
Option B:	finite
Option C:	uncountable
Option D:	Both countable as well as uncountable

6.	$f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2$ for $x \in \mathbb{R}$ then $f$ is
Option A:	injective
Option B:	surjective
Option C:	bijjective
Option D:	not bijective
7.	$f(x) = x + 3$ $g(x) = 2x + 1$ then $g \circ f(x) =$
Option A:	$2x - 7$
Option B:	$2x + 7$
Option C:	$2x + 4$
Option D:	$3x + 4$
8.	$L\{t \sin t\} =$
Option A:	$\frac{2s}{(s^2 + 1)^2}$
Option B:	$\frac{-2s}{(s^2 + 1)^2}$
Option C:	$\frac{s}{(s^2 + 1)^2}$
Option D:	$\frac{1}{(s^2 + 1)^2}$
9.	Inverse Laplace transform of $\frac{1}{s(s+1)}$ is
Option A:	$1 - e^{-t}$
Option B:	$1 - e^t$
Option C:	$\cos ht$
Option D:	$e^{-t}$
10.	If $f(z) = \bar{z}$ where $z = x + iy$ then which of the following is true ?
Option A:	$f(z)$ is everywhere analytic
Option B:	Cauchy-Riemann equations are satisfied
Option C:	$f(z)$ is not analytic at $x = 0$
Option D:	$f(z)$ is analytic only at $x = 0$
11.	Fixed points of transformation $f(z) = \frac{z-1}{z+1}$ are
Option A:	$\pm 1$
Option B:	$\pm i$
Option C:	$\pm 2i$
Option D:	$\pm 2$
12.	How many friends you must have to gurantee that at least two of them have birthday in same month
Option A:	8
Option B:	13
Option C:	12
Option D:	10
13.	Analytic function $f(z) = u + iv$ whose imaginary part $v = \tan^{-1} \frac{y}{x}$ is



Option A:	$\tan z$
Option B:	$\log z$
Option C:	$\sin z$
Option D:	$\cos z$
14.	A relation R is defined on $\mathbb{Z}$ such that $aRb$ if $a - b$ is divisible by 5. How many distinct equivalence classes are there corresponding to R?
Option A:	1
Option B:	3
Option C:	4
Option D:	5
15.	$L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$ then $L\{J_0(4t)\} =$
Option A:	$\frac{1}{\sqrt{s^2+16}}$
Option B:	$\frac{4}{\sqrt{s^2+16}}$
Option C:	$\frac{4}{\sqrt{s^2+4}}$
Option D:	$\frac{1}{4\sqrt{s^2+16}}$
16.	Image of $ z  = 1$ under $w = z + 2 + 3i$ is
Option A:	straight line
Option B:	line segment
Option C:	circle
Option D:	ellipse
17.	If repetitions are not permitted, How many 4-digit numbers can be formed using digits 1,2,3,5,7,8
Option A:	360
Option B:	720
Option C:	180
Option D:	1296
18.	From integers 1 to 100, any one integer is chosen at random. Determine probability that it is divisible by 3 or 5.
Option A:	0.47
Option B:	0.53
Option C:	0.59
Option D:	0.48
19.	$P(A) = \frac{1}{2}$ , $P(B) = \frac{1}{3}$ where A and B are independent events then $P(A \cup B) =$
Option A:	$\frac{2}{3}$
Option B:	$\frac{1}{3}$
Option C:	$\frac{1}{6}$

Option D:	$\frac{5}{6}$
20.	Three students solve a problem in Mathematics independently. Their chances of solving problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Probability that problem is solved is
Option A:	$\frac{1}{4}$
Option B:	$\frac{3}{4}$
Option C:	$\frac{1}{24}$
Option D:	$\frac{13}{12}$

<b>Q2.</b> (20 Marks)	<b>Solve any Four out of Six. (5 marks each)</b>
A	Determine constants a,b,c,d so that $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic
B	$f: \mathbb{R} \rightarrow \mathbb{R} \quad g: \mathbb{R} \rightarrow \mathbb{R} \quad h: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = x + 4, g(x) = x - 4, h(x) = 4x$ for $x \in \mathbb{R}$ Compute $f \circ g, g \circ f, h \circ h$
C	Find $L\{te^{3t} \sin 4t\}$
D	Find $L^{-1}\left\{\frac{s+2}{(s^2+4s+8)^2}\right\}$
E	In a bolt factory, machines A, B, C manufacture respectively 25%, 35% and 40% of total production. Of this output, Defective bolts produced by machine A, B, C are 5%, 4% and 3% respectively. A bolt is drawn at random from total production and is found to be defective. What is the probability that it is manufactured by machine A?
F	If four points are drawn inside an equilateral triangle of side 1 unit then prove that there are two among them whose distance apart is less than $\frac{1}{2}$ units.

<b>Q3.</b> (20 Marks)	<b>Solve any Four out of Six. (5 marks each)</b>
A	Find $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$
B	Evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$
C	$f: \mathbb{R} - \left\{\frac{7}{3}\right\} \rightarrow \mathbb{R} - \left\{\frac{4}{3}\right\} \quad f(x) = \frac{4x-5}{3x-7}$ Prove that $f$ is bijective. Hence find $f^{-1}$
D	Find bilinear transformation which maps points $2, i, -2$ in Z-plane onto points $1, i, -1$ in W-plane.
E	Construct analytic function $f(z) = u + iv$ where $v = e^x(x \sin y + y \cos y)$
F	A student giving true false test answers a question correctly if he knows the answer and if he <b>does not know</b> the answer then he answers a question on basis of tossing a coin. If probability that student knows the answer is $\frac{1}{5}$ then what is the probability that students knows the answer to a correctly

