University of Mumbai

Examinations Summer 2022

Time: 2 hour 30 minutes Max. Marks: 80

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Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	$L\{t^3 + e^{-3t}\}$ equals
Option A:	$\frac{6}{s^4} + \frac{1}{s+3}$
Option B:	$\frac{6}{s^4} - \frac{1}{s-3}$
Option C:	$\frac{4!}{s^4} + \frac{1}{s+3}$
Option D:	$\frac{4!}{s^4} + \frac{1}{s-3}$
2.	$L\{e^{2t}\cos 3t\}$ equals
Option A:	$\frac{s+2}{(s+2)^2+9}$
Option B:	$\frac{3}{(s+2)^2+9}$
Option C:	$\frac{s-2}{(s-2)^2+9}$
Option D:	$\frac{3}{(s-2)^2+9}$
3.	$L^{-1}\{\frac{2}{(s-2)(s+3)}\}$ equals
Option A:	$\frac{2}{5}(e^{-2t} - e^{3t})$
Option B:	$\frac{2}{5}(e^{2t} - e^{-3t})$
Option C:	$2(e^{2t} - e^{-3t})$
Option D:	$\frac{2}{5}(e^{2t} + e^{-3t})$
4.	$L^{-1}\{\frac{s-1}{s^2-2s+5}\}$ equals
Option A:	$e^{-t}\cos 2t$
Option B:	$e^{t}\sin 2t$
Option C:	$e^{-t}\sin 2t$
Option D:	$e^{t}\cos 2t$
5.	If $u = y^2 - 2xy + ax^2 + 5x - 3y$ is harmonic, then
Option A:	$a = 1$
Option B:	$a = -1$
Option C:	$a = -2$
Option D:	$a = 2$
6.	The function $f(z) = \frac{z}{(z-3)^2(z+i)^3}$ has
Option A:	Poles of order 3 and 2 respectively at $z = -i$ and $z = 3$

Option B:	Poles of order 2 and 3 respectively at $z = -3$ and $z = i$
Option C:	Poles of order 3 and 2 respectively at $z = i$ and $z = -3$
Option D:	Poles of order 2 and 3 respectively at $z = 3$ and $z = i$
7.	Suppose $f(x) = x$ in $(0,2)$. Then the Fourier coefficient a_0, where $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$ is the Fourier Series of $f(x)$ is equal to
Option A:	0
Option B:	1
Option C:	2
Option D:	-1
8.	The functions $f(x) = 1$ and $g(x) = x$ are defined in the interval (-1,1). Then
Option A:	$f(x)$ and $g(x)$ are orthonormal in (-1,1)
Option B:	$f(x)$ and $g(x)$ are orthogonal, but not orthonormal in (-1,1)
Option C:	$f(x)$ and $g(x)$ are not orthogonal in (-1,1)
Option D:	$f(x)$ and $g(x)$ are orthonormal, but not orthogonal in (-1,1)
9.	Identify the one- dimensional wave equation among the following:
Option A:	$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
Option B:	$5\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$
Option C:	$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$
Option D:	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
10.	Suppose the two regression coefficients are $b_{yx} = \frac{-1}{2}, b_{xy} = \frac{-1}{4}$ then the correlation coefficient r is
Option A:	$-\sqrt{\frac{1}{8}}$
Option B:	$\pm\sqrt{\frac{1}{8}}$
Option C:	$\sqrt{\frac{1}{8}}$
Option D:	$\frac{1}{8}$

Subjective/Descriptive questions

Q2 **(20 Marks)**	**Solve any Four out of Six (5 marks each)**
A	Evaluate using Laplace Transforms:$\int_0^\infty e^{-4t} cos\, 2t \sin 5t\, dt$
B	Find $L^{-1}\{\frac{s}{(\,(s^2-8s+25)}\}$
C	Find a, b, c, d if $f(z) = (x^3 + axy^2 + by^2 + x^2) + i(cx^2y - y^3 + dxy)$ is analytic.
D	Suppose $f(a) = \int_C \frac{2z^2-3z+7}{z-a}\, dz$ where a lies inside C and C is the circle $\|z-1\| = 2.$ Evaluate $f(2)$ and $f'(2)$
E	Obtain the Fourier series of $f(x) = x^3 ,\ -\pi \le x \le \pi$
F	Solve using Bender-Schmidt method:$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$; subject to the conditions: $u(0,t) = 0; u(4,t) = 0; u(x,0) = x^2(16 - x^2)$ taking $h = 1$ upto 2 seconds

<table>
<tr><th>Q3
(20 Marks)</th><th>Solve any Four out of Six (5 marks each)</th></tr>
<tr><td>A</td><td>Obtain $L\{\int_0^t u \sin 2u\ du\}$</td></tr>
<tr><td>B</td><td>Obtain the analytic function whose real part is $e^{-x}\cos y$.</td></tr>
<tr><td>C</td><td>Fit a straight line to the following data by the method of least squares where Y is the dependent variable:
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Y</td><td>12.2</td><td>13.4</td><td>11.8</td><td>14.6.</td><td>14.8</td><td>16.2</td></tr>
</table></td></tr>
<tr><td>D</td><td>Evaluate $\int_C \frac{z}{(z-4)(z+2)}\, dz$ using Cauchy's Residue Theorem where C is the circle $|z-3| = 2$</td></tr>
<tr><td>E</td><td>Obtain the Half Range Fourier Cosine series of
$f(x) = x - x^2 ,\ 0 < x < 1$</td></tr>
<tr><td>F</td><td>Solve using Crank-Nicolson formula: $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0,\ 0 \le x \le 1$; subject to the conditions: $u(0,t) = 0; u(1,t) = 100\, t; u(x,0) = 0$ taking $h = 0.25$ for one time-step</td></tr>
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<tr><th>Q4
(20 Marks)</th><th>Solve any Four out of Six (5 marks each)</th></tr>
<tr><td>A</td><td>Find: $L^{-1}\{\frac{s}{(s^2+4)(s^2+1)}\}$ using convolution theorem</td></tr>
<tr><td>B</td><td>Obtain the Bilinear transformation that transforms the points $z = -1,0,1$ respectively to the points $w = \infty, -1, 0$</td></tr>
<tr><td>C</td><td>Obtain the Laurent series for the function $f(z) = \frac{1}{z(z-1)}$ around $z = 0$ in the region $|z| > 1$</td></tr>
<tr><td>D</td><td>Obtain the complex form of Fourier series of $f(x) = e^{4x}$ in $(0, 4)$</td></tr>
<tr><td>E</td><td>Obtain the Spearman’s rank correlation coefficient of the following marks in Subjects X and Y :
<table>
<tr><td>X</td><td>20</td><td>15</td><td>18</td><td>15</td><td>15</td><td>12</td></tr>
<tr><td>Y</td><td>16</td><td>19</td><td>16</td><td>14</td><td>17</td><td>14</td></tr>
</table></td></tr>
<tr><td>F</td><td>Obtain the solution of the one – dimensional heat equation: $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
using the boundary conditions:
$u(0,t) = 0; u(l,t) = 0; u(x,0) = x, \ 0 < x < l$ where l is the length of the rod</td></tr>
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